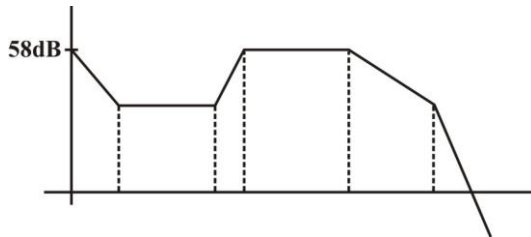


Sol. 1(a)

$a = 1/4$        $b = 1/24$



$$58 - 40 = 20 \log k$$

$$K = 7.95$$

$$\frac{a}{bk} = \frac{6}{7.95} = 0.755 \quad \text{ANS}$$

Sol.1(b)

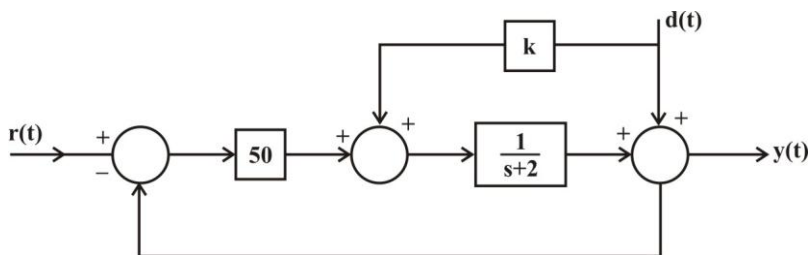
$$G(s) = \frac{k_i e^{-s}}{s}$$

$$1 + G(s) = \frac{s + k_i e^{-s}}{s}$$

$$\frac{C}{R} = \frac{k_i e^{-s}}{s + k_i e^{-s}}$$

$$k_i e^{+1} = 1 \quad k_i = \frac{1}{e} = 0.367 \quad \text{ANS}$$

Sol.1(c)

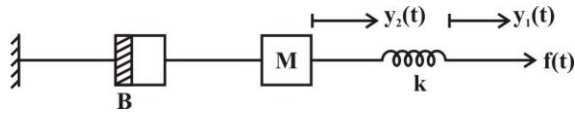


$$\frac{Y(s)}{D(s)} = \frac{1 + \frac{k}{s+2}}{1 - \left[ \frac{50}{(s+2)} (-1) \right]} = 0$$

$$1 + \frac{k}{s+2} = 0 \quad \therefore k = -(s+2)$$

$$\text{Zero mean } 1 + \frac{k}{s+2} = 0 \quad \therefore k = -(s+2) \text{ at } s=0 \quad k = -2 \quad \text{ANS}$$

Sol. 1(d)

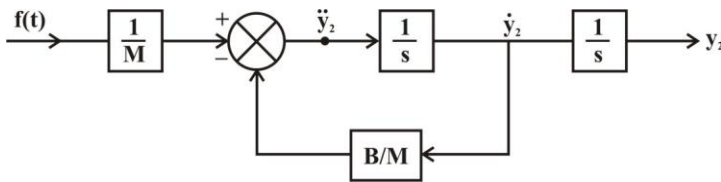


$$M\ddot{y}_2 + B\dot{y}_2 + k(y_2 - y_1) = 0 \quad \text{--- (1)}$$

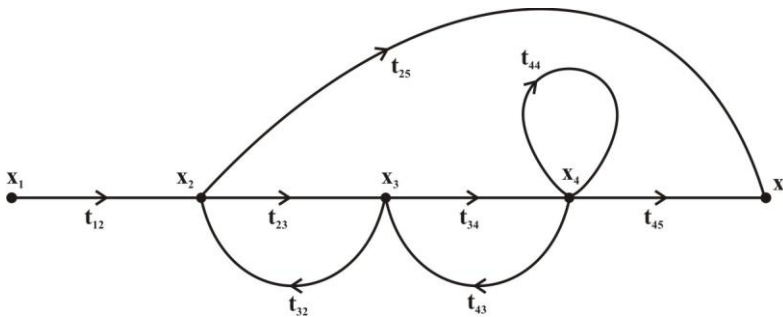
$$f(t) = k(y_1 - y_2) \quad \text{--- (2)}$$

$$M\ddot{y}_2 + B\dot{y}_2 = f(t)$$

$$\ddot{y}_2 = -\frac{B}{m}\dot{y}_2 - \frac{f(t)}{m}$$



Sol.2 (a)



Sol.2 (b) 
$$\Delta = \left( 1 + \frac{p_1}{s} + \frac{p_2}{s} + \frac{p_1 p_2}{s^2} \right)$$

$$p_1 = \frac{a_0}{b_0} \frac{z_1 z_2}{s^2}$$

$$p_2 = \frac{a_0}{b_0} \frac{z_1}{s}$$

$$p_3 = \frac{a_0}{b_0} \frac{z_2}{s}$$

$$p_4 = \frac{a_0}{b_0}$$

Sol. 2(c) 
$$G(s)H(s) = \frac{s}{(s+100)^3}$$

$$\phi = 90^\circ - 3 \tan^{-1}(\omega/100)$$

For PCF  $\phi = -180^\circ$

$$\therefore -180^\circ = 90^\circ - 3 \tan^{-1}(\omega/100)$$

$$\tan^{-1}(\omega/100) = 90^\circ \Rightarrow (\omega = \infty)$$

$\therefore$  PCF is  $\infty$ , so (GM =  $\infty$ )

For calculation of GCF,  $|G(s)H(s)| = 1$  So  $\omega$  is not defined hence (PM =  $\infty$ ) **ANS**

**Sol. 3(a)** Characteristic equation is  $1+G(s)H(s)=0$

$$\therefore s^3 + as^2 + 2s + 1 + k(s+1) = 0$$

$$s^3 + as^2 + s(k+2) + (k+1) = 0$$

$$s^3 \quad 1 \quad k+2$$

$$s^2 \quad a \quad k+1$$

$$s^1 \quad (k+2) - \frac{k+1}{a} \quad 0$$

$$s^0 \quad k+1 \quad 0$$

For oscillations all elements in  $s^1$  row should be zero.

$$\text{So } k+2 = \frac{k+1}{a} \quad \text{--- (1)}$$

$$\text{But } as^2 + (k+1) = 0$$

$$\omega_n = 2 \text{ Rad/sec}$$

$$\therefore k+1 = 4a \quad \text{--- (2)}$$

$$\text{By equation (1) \& (2) } \quad k=2, a=0.75 \text{ ANS}$$

**Sol.3 (b)**

$$G(s) = \frac{0.4s+1}{s(s+0.6)}$$

$$s^2 + 0.6s + 0.4s + 1 = 0$$

$$s^2 + s + 1 = 0$$

$$\omega_n = 1, \text{ Rad/sec, } (\xi = 0.5)$$

$$(1) \quad M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.16$$

$$(2) \quad \omega_d = \omega_n \sqrt{1-\xi^2} = 0.866$$

$$(3) \quad t_s = \frac{4}{\xi\omega_n} = 8 \text{ sec}$$

$$(4) \quad C(t) = 1 - e^{-\frac{\xi\omega_n t}{\sqrt{1-\xi^2}}} \sin(\omega_d t + \phi) \quad \text{(b) Peak overshoot} = M_p = 16.3\%$$

$$\omega_d = 0.86$$

$$T_s = \frac{4}{\xi\omega_n} = 8 \text{ sec}$$

**Sol.3(c)** Phase delay  $(t_d) = -\frac{d\phi}{d\omega} \quad t_d = -\left(\frac{1}{1+\omega^2}\right)$

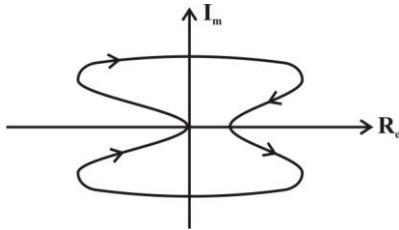
Avg. value of  $t_d$  over one frequency range  $0 \leq \omega \leq 10$  is

$$\langle t_d \rangle = \frac{1}{10} \int_0^{10} \frac{d\omega}{1+\omega^2} = \frac{1}{10} \tan^{-1}(10)$$

$$= \frac{1}{10} \times 84.2 \times \frac{\pi}{180} = 0.147 \text{ sec } \text{ANS}$$

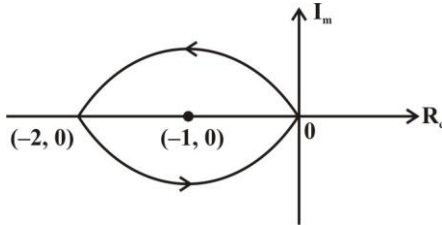
**Sol.4 (a)**

$$G(s)H(s) = \frac{1}{s^2 + a^2}$$



**Sol.4(b)**

Nyquist criterion graph for given function is



Here  $N = 1 \rightarrow$  &  $P = 1 \Rightarrow \boxed{z = 0}$

So system is stable. **ANS**

**Sol.4(c)**

If  $r(t) = 40 + 20t + 5t^2$

$$e_{ss} = \frac{40}{1+k_p} + \frac{20}{k_v} + \frac{2 \times 5}{k_a}$$

$$k_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty$$

$$k_v = \lim_{s \rightarrow 0} sG(s)H(s) = \infty$$

$$k_a = 2.5 \quad \therefore \quad e_{ss} = \frac{10}{2.5} = 4 \text{ ANS}$$

**Sol.5 (a)**

(1) No. of asymptotes =  $N = P - z = 3 - 0 = 3$

$$C = \frac{\sum P - \sum Z}{P - z} = -1.66$$

$$\text{Angle of asymptotes} = \frac{(2k+1)\pi}{P-z} = 60^\circ, 180^\circ, 300^\circ$$

Break away point  $\frac{dk}{ds} = 0$

$$\Rightarrow s = -0.46, -2.86$$

Here  $s = -0.46$  is valid break away

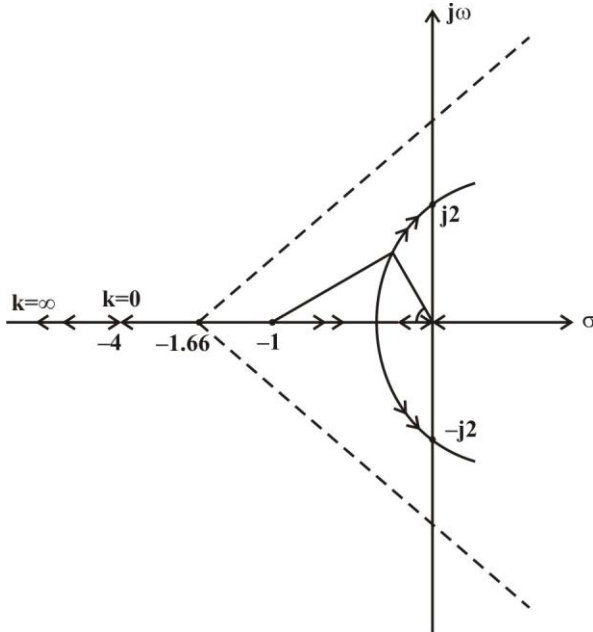
Interaction of Root Locus with Imaginary axis can be calculated by  $s^3 + 5s^2 + 4s + k = 0$

$$\begin{array}{rcl} s^3 & 1 & 4 \\ s^2 & 5 & k \\ s^1 & \frac{20-k}{5} & 0 \\ s^0 & k & 0 \end{array}$$

For oscillations :  $5s^2 + k = 0$

$$k = 20, \Rightarrow \omega_n = 2 \text{ Rad/sec}$$

at  $k = 20$ , system is marginally stable.



Here  $\cos^{-1} \xi = 70^\circ \therefore \xi = 0.34$  **ANS**

**Sol.5(b)** Transfer function of II order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s_1 = -\xi\omega_n \pm \omega_n \sqrt{\xi^2 - 1}$$

There will 3 cases acc. to value of  $\xi$ .

$\xi = 1$ , critical damped

$\xi < 1$ , under damped

$\xi > 1$ , over damped

For under damped system:  $(0 < \xi < 1)$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \xi\omega_n + j\omega_d)(s + \xi\omega_n - j\omega_d)}$$

if  $r(t) = u(t) \rightarrow R(s) = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s(s + \xi\omega_n + j\omega_d)(s + \xi\omega_n - j\omega_d)}$$

Two poles are  $s_1 = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

$$\begin{aligned} C(s) &= \frac{1}{s} - \frac{(s + 2\xi\omega_n)}{(s^2 + 2\xi\omega_n s + \omega_n^2)} \\ &= \frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \end{aligned}$$

$$\begin{aligned} C(t) &= 1 - e^{-\xi\omega_n t} \left( \cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t \right) \\ &= 1 - e^{-\xi\omega_n t} \sin(\omega_d t + \phi) \end{aligned}$$

**Sol.5(c)**

The dynamic error may be evaluated using dynamic error coefficients, the concept is generalized to include inputs of any arbitrary uncton of time.

For a unity feedback express  $N^r$  and  $D^r$  of

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

In ascending form of S and performing long divisions we have

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)} = C_0 + C_1 S + \frac{C_2 S^2}{|2} + \dots$$

here  $C_0, C_1, C_2, C_3, \dots$  are known as dynamic error coefficient or generalized error coefficients

$$\rightarrow E(s) = C_0 R(s) + C_1 S R(s) + \frac{C_2 S^2}{|2} R(s) + \dots$$

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{|2} \ddot{r}(t) + \dots$$

Generalized error coefficients are

$$C_0 = \lim_{s \rightarrow 0} F(s)$$

$$C_1 = \lim_{s \rightarrow 0} \frac{d}{ds} F(s)$$

$$C_2 = \lim_{s \rightarrow 0} \frac{d^2}{ds^2} F(s)$$

$$C_n = \lim_{s \rightarrow 0} \frac{d^n}{ds^n} F(s)$$

**Sol.6(a)** Characteristic equation is  $1 + G(s)H(s) = 0$

$$s^2 + 3 + k(s^2 - 4) = 0$$

$$s^2(1+k) + (3k-4) = 0$$

$$s^2 = \frac{-(3k-4)}{(k+1)}$$

For stable,  $k < 3/4$  &  $k > -1$  So  $k \in (-1, 3/4)$  **ANS**

**Sol.6 (b)** Characteristic equation is  $1 + \frac{k}{s(s+a)} = 0$

$$s^2 + as + k = 0$$

$$M_r = \frac{1}{2\xi\sqrt{1-2\xi^2}} = 1.04$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} = 11.55$$

Here  $\omega_n = \sqrt{k}$ ,  $2\xi\omega_n = a$

So here  $k = 476.11$  **ANS**

$a = 26.192$  **ANS**

**Sol.6(c)** (i) Transfer function is  $C[SI - A]^{-1}B$  T/F function =  $\frac{s+3}{s^2+3s+2}$

(ii)  $|SI - A| = 0 \rightarrow$  Eigen values  $s = -1, -2$

(iii) 
$$\begin{bmatrix} e^{-t} & 0 \\ e^{-t} - e^{-2t} & e^{-2t} \end{bmatrix}$$

(iv) Put  $\dot{X}(t) = AX(t) + Bu(t)$

Let  $X(t) = kz(t)$

$$\dot{X}(t) = k\dot{z}(t)$$

$$k\dot{z}(t) = Akz(t) + Bu(t)$$

$$\dot{z}(t) = k^{-1}Akz(t) + A^{-1}Bu(t)$$

$$z(t) = M(z(t)) + Nu(t)$$

**Sol.7 (a)** Characteristic equation is  $1 + G(s)H(s) = 0$

$$s^2 + 3 + k(s^2 - 4) = 0$$

$$s^2(1+k) + (3k-4) = 0$$

$$s^2 = \frac{-(3k-4)}{(k+1)}$$

For stable,  $k < 3/4$  &  $k > -1$  so  $k \in (-1, 3/4)$  **ANS**

**Sol.7 (b)**

$$\phi = -\tan^{-1} \omega - \tan^{-1} (\omega/2)$$

$$\phi = -90^\circ = -\tan^{-1} \omega - \tan^{-1} \omega/2$$

$$\therefore \omega^2 = 2 \Rightarrow \omega = \sqrt{2} \text{ Rad/sec}$$

if K/S is placed in forward path

$$G(s) = \frac{k/s}{(s+1)(s+2)}$$

$$\text{For G.M} \rightarrow \phi = -90^\circ - \tan^{-1} \omega - \tan^{-1} (\omega/2)$$

$$\therefore \phi = -180^\circ \rightarrow \omega = \sqrt{2} \rightarrow (\text{PCF}) \text{ **ANS**}$$

$$\rightarrow G(s)H(s) = \frac{k}{\sqrt{2} \times \sqrt{(1+2)(4+2)}} = \frac{k}{6}$$

$$\text{GM} = \frac{6}{k} = 2.5 \Rightarrow (k = 2.4) \text{ **ANS**}$$

**Sol.7(c)**

$$T(s) = \frac{10^8 (s+0.1)^3}{(s+10)^2 (s+100)}$$

$$\frac{k \left(1 + \frac{s}{0.1}\right)^3}{\left(1 + \frac{s}{10}\right)^2 \left(\frac{s}{100} + 1\right)} = \frac{k (s+0.1)^3 \times 10^3 \times 10^4}{(s+10)^2 \times (s+10)} = \frac{k \times 10^7 (s+0.1)^3}{(s+10)^2 (s+10)} \text{ (here } k=1) \text{ **ANS**}$$