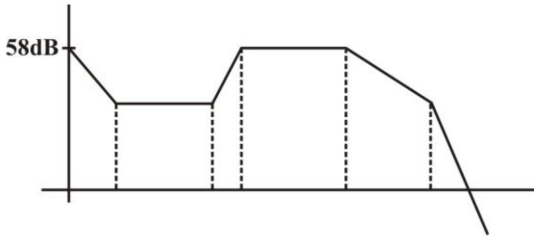
Control Conventional Solution (ESE-2015 Test Series Dated 01.05.2015)

Sol. 1(a) $a = 1/4 \qquad b = 1/24$



$$58 - 40 = 20\log k$$

$$K = 7.95$$

$$\frac{a}{bk} = \frac{6}{7.95} = 0.755 \quad \textbf{ANS}$$

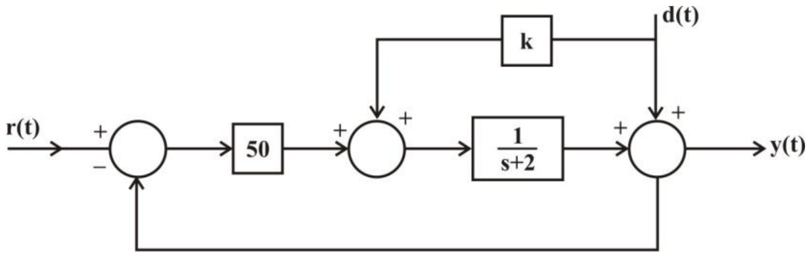
Sol.1(b) $G(s) = \dfrac{k_i e^{-s}}{s}$

$$1 + G(s) = \frac{s + k_i e^{-s}}{s}$$

$$\frac{C}{R} = \frac{k_i e^{-s}}{s + k_i e^{-s}}$$

$$k_i e^{+1} = 1 \quad k_i = \frac{1}{e} = 0.367 \quad \textbf{ANS}$$

Sol.1(c)



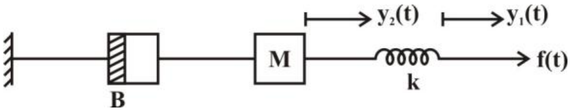
$$\frac{Y(s)}{D(s)} = \frac{1 + \dfrac{k}{s+2}}{1 - \left[\dfrac{50}{(s+2)}(-1)\right]} = 0$$

$$1 + \frac{k}{s+2} = 0 \quad \therefore \quad k = -(s+2)$$

Zero mean $1 + \dfrac{k}{s+2} = 0 \quad \therefore \quad k = -(s+2)$ at $s=0$ $k = -2$ **ANS**

28A/11 Ground Floor Jia Sarai (Near IIT)- Hauz Khas Metro Station New Delhi-110016
Helpline No.: 09811382221, 011-32662945
Website: www.panaceainstitute.org.in, E-mail: panacea.institute@gmail.com

Sol. 1(d)

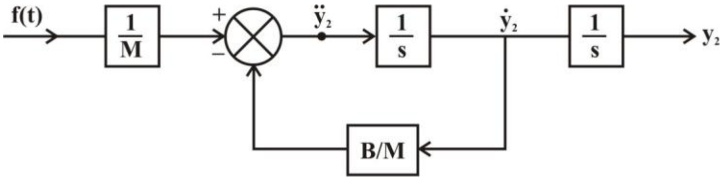


$$M\ddot{y}_2 + B\dot{y}_2 + k(y_2 - y_1) = 0 \qquad \text{— (1)}$$

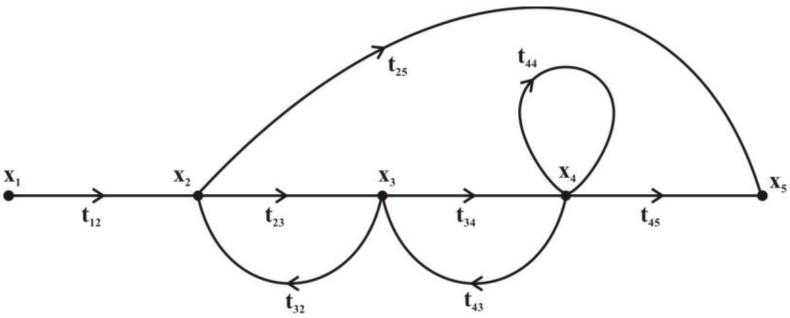
$$f(t) = k(y_1 - y_2) \qquad \text{— (2)}$$

$$M\ddot{y}_2 + B\dot{y}_2 = f(t)$$

$$\ddot{y}_2 = -\frac{B}{m}y_2 - \frac{f(t)}{m}$$



Sol.2 (a)



Sol.2 (b) $\Delta = \left(1 + \frac{p_1}{s} + \frac{p_2}{s} + \frac{p_1 p_2}{s^2}\right)$

$p_1 = \frac{a_0}{b_0}\frac{z_1 z_2}{s^2}$ $\qquad p_2 = \frac{a_0}{b_0}\frac{z_1}{s}$ $\qquad P_3 = \frac{a_0}{b_0}\frac{z_2}{s}$ $\qquad p_4 = \frac{a_0}{b_0}$

Sol. 2(c) $G(s)H(s) = \frac{s}{(s+100)^3}$

$$\phi = 90^o - 3\tan^{-1}(\omega/100)$$

For PCF $\phi = -180^o$

$\therefore \qquad -180^o = 90^o - 3\tan^{-1}(\omega/100)$

$\qquad \tan^{-1}(\omega/100) = 90^o \quad \Rightarrow \quad (\omega = \infty)$

$\because \qquad$ PCF is ∞, so $(GM = \infty)$

For calculation of GCF, $|G(s)H(s)| = 1$ $\qquad$ So ω is not defined hence (PM = ∞) **ANS**

Sol. 3(a) Characteristic equation is $1+G(s)H(s)=0$

$\therefore \quad s^3+as^2+2s+1+k(s+1)=0$

$$s^3+as^2+s(k+2)+(k+1)=0$$

s^3	1	$k+2$
s^2	a	$k+1$
s^1	$(k+2)-\frac{k+1}{a}$	0
s^0	$k+1$	0

For oscillations all elements in s^1 row should be zero.

So $\quad k+2=\frac{k+1}{a} \qquad$ — (1)

But $\quad as^2+(k+1)=0$

$$\omega_n = 2\,\text{Rad/sec}$$

$\therefore \quad k+1=4a \qquad$ — (2)

By equation (1) & (2) $\quad k=2,\ a=0.75$ **ANS**

Sol.3 (b) $G(s)=\frac{0.4s+1}{s(s+0.6)}$

$$s^2+0.6s+0.4s+1=0$$

$$s^2+s+1=0$$

$$\omega_n=1,\ \text{Rad/sec},(\xi=0.5)$$

(1) $\quad M_p=e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}=0.16$

(2) $\quad \omega_d=\omega_n\sqrt{1-\xi^2}=0.866$

(3) $\quad t_s=\frac{4}{\xi\omega_n}=8\,\text{sec}$

(4) $\quad C(t)=1-e^{-\frac{\xi\omega_n t}{\sqrt{1-\xi^2}}}\sin(\omega_d t+\phi)$ (b) $\quad$ Peak overshoot $=M_p=16.3\%$

$$\omega_d=0.86$$

$$T_s=\frac{4}{\xi\omega_n}=8\,\text{sec}$$

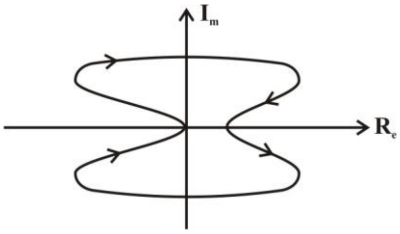
Sol.3(c) Phase delay $\quad (t_d)=-\frac{d\phi}{d\omega} \qquad t_d=-\left(\frac{1}{1+\omega^2}\right)$

Avg. value of t_d over one frequency range $0\le\omega\le10$ is

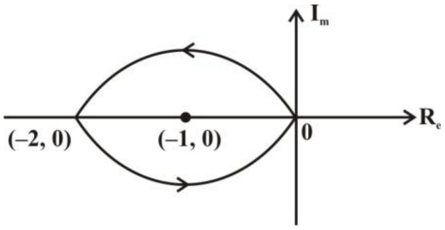
$$\langle t_d\rangle=\frac{1}{10}\int_0^{10}\frac{d\omega}{1+\omega^2}=\frac{1}{10}\tan^{-1}(10)$$

$$= \frac{1}{10} \times 84.2 \times \frac{\pi}{180} = 0.147 \sec \text{ **ANS**}$$

Sol.4 (a) $G(s)H(s) = \frac{1}{s^2 + a^2}$



Sol.4(b) Nyquist criterion graph for given function is



Here $N = 1 \rightarrow$ & $P = 1$ $\Rightarrow$ $\boxed{z = 0}$

So system is stable. **ANS**

Sol.4(c) If $r(t) = 40 + 20t + 5t^2$

$$e_{ss} = \frac{40}{1 + k_p} + \frac{20}{k_v} + \frac{2 \times 5}{k_a}$$

$$k_p = \lim_{s \to 0} G(s)H(s) = \infty$$

$$k_v = \lim_{s \to 0} sG(s)H(s) = \infty$$

$k_a = 2.5$ $\therefore$ $e_{ss} = \frac{10}{2.5} = 4$ **ANS**

Sol.5 (a)

(1) No. of asymptotes $= N = P - z = 3 - 0 = 3$

$$C = \frac{\Sigma P - \Sigma z}{P - z} = -1.66$$

Angle of asymptotes $= \frac{(2k+1)\pi}{P - z} = 60^\circ, 180^\circ, 300^\circ$

Break away point $\frac{dk}{ds} = 0$

$\Rightarrow$ $s = -0.46, -2.86$

Here $s = -0.46$ is valid break away

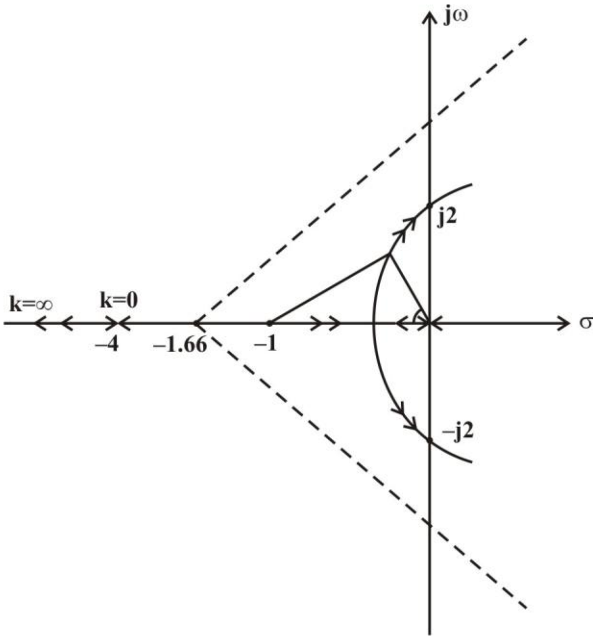
Interaction of Root Locus with Imaginary axis can be calculated by $s^3 + 5s^2 + 4s + k = 0$

s^3	1	4
s^2	5	k
s^1	$\frac{20-k}{5}$	0
s^0	k	0

For oscillations : $5s^2 + k = 0$

$k = 20, \quad \Rightarrow \quad \omega_n = 2\,\text{Rad/sec}$

at k = 20, system is marginally stable.



Here $\cos^{-1}\xi = 70^o \quad \therefore \quad \boxed{\xi = 0.34}$ **ANS**

Sol.5(b) Transfer function of II order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^{\ 2}}{s^2 + 2\xi\omega_n s + \omega_n^{\ 2}}$$

$$s_1 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

There will 3 cases acc. to value of ξ.

$\xi = 1$, critical damped

$\xi < 1$, under damped

$\xi > 1$, our damped

For under damped system: $(0 < \xi < 1)$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^{\ 2}}{(s + \xi\omega_n + j\omega_d)(s + \xi\omega_n - j\omega_d)}$$

if $\quad r(t) = u(t) \rightarrow R(s) = \frac{1}{s}$

$$C(s)=\frac{\omega_n^{\,2}}{s\left(s+\xi\omega_n+j\omega_d\right)\left(s+\xi\omega_n-j\omega_d\right)}$$

Two poles are $s_1=-\xi\omega_n\pm j\omega_n\sqrt{1-\xi^2}$

$$C(s)=\frac{1}{s}-\frac{\left(s+2\xi\omega_n\right)}{\left(s^2+2\xi\omega_n s+\omega_n^{\,2}\right)}$$

$$=\frac{1}{s}-\frac{\left(s+\xi\omega_n\right)}{\left(s+\xi\omega_n\right)^2+\omega_d^{\,2}}-\frac{\xi\omega_n}{\left(s+\xi\omega_n\right)^2+\omega_d^{\,2}}$$

$$C(t)=1-e^{-\xi\omega_n t}\left(\cos\left(\omega_d t\right)+\frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$=1-e^{-\xi\omega_n t}\sin\left(\omega_d t+\phi\right)$$

Sol.5(c) The dynamic error may be evaluated using dynamic error coefficients, the concept is generalized to include inputs of any arbitrary unction of time.

For a unity feedback express N^r and D^r of

$$\frac{E(s)}{R(s)}=\frac{1}{1+G(s)H(s)}$$

In ascending form of S and performing long divisions we have

$$\frac{E(s)}{R(s)}=\frac{1}{1+G(s)}=C_0+C_1S+\frac{C_2S^2}{\lfloor 2}+----$$

here $C_0, C_1, C_2, C_3 ----$ are known as dynamic error coefficient or generalized error coefficients

$$\rightarrow \quad E(s)=C_0R(s)+C_1SR(s)+\frac{C_2S^2}{\lfloor 2}R(s)+----$$

$$e(t)=C_0r(t)+C_1\dot{r}(t)+\frac{C_2}{\lfloor 2}\ddot{r}(t)+----$$

Generalized error coefficients are

$$C_0=\lim_{s\to 0}F(s)$$

$$C_1=\lim_{s\to 0}\frac{d}{ds}F(s)$$

$$C_2=\lim_{s\to 0}\frac{d^2}{ds^2}F(s)$$

$$C_n=\lim_{s\to 0}\frac{d^n}{ds^n}F(s)$$

Sol.6(a) Characteristic equation is $1+G(s)H(s)=0$

$$s^2+3+k(s^2-4)=0$$

$$s^2(1+k)+(3k-4)=0$$

$$s^2=\frac{-(3k-4)}{(k+1)}$$

For stable, $k<3/4$ & $k>-1$ So $k\varepsilon(-1,3/4)$ **ANS**

Sol.6 (b) Characteristic equation is $1+\frac{k}{s(s+a)}=0$

$$s^2+as+k=0$$

$$M_r=\frac{1}{2\xi\sqrt{1-2\xi^2}}=1.04$$

$$\omega_r=\omega_n\sqrt{1-2\xi^2}=11.55$$

Here $\omega_n=\sqrt{k}$, $2\xi\omega_n=a$

So here $k=476.11$ **ANS**

$a=26.192$ **ANS**

Sol.6(c) (i) Transfer function is $C[SI-A]^{-1}B$ T/F function $=\frac{s+3}{s^2+3s+2}$

(ii) $|SI-A|=0\rightarrow$ Eigen values $s=-1,-2$

(iii) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t}-e^{-2t} & e^{-2t} \end{bmatrix}$

(iv) Put $\dot{X}(t)=AX(t)+Bu(t)$

Let $X(t)=kz(t)$

$$\dot{X}(t)=k\dot{z}(t)$$

$$k\dot{z}(t)=Akz(t)+Bu(t)$$

$$\dot{z}(t)=k^{-1}Akz(t)+A^{-1}Bu(t)$$

$$z(t)=M(z(t))+Nu(t)$$

Sol.7 (a) Characteristic equation is $1+G(s)H(s)=0$

$$s^2+3+k(s^2-4)=0$$

$$s^2(1+k)+(3k-4)=0$$

$$s^2=\frac{-(3k-4)}{(k+1)}$$

For stable, $k<3/4$ & $k>-1$ so $k\varepsilon(-1,3/4)$ **ANS**

Sol.7 (b) $\phi = -\tan^{-1}\omega - \tan^{-1}(\omega/2)$

$\phi = -90^o = -\tan^{-1}\omega - \tan^{-1}\omega/2$

$\therefore \quad \omega^2 = 2 \Rightarrow \omega = \sqrt{2}$ Rad/sec

if K/S is placed in forward path

$$G(s) = \frac{k/s}{(s+1)(s+2)}$$

For G.M $\rightarrow \phi = -90^o - \tan^{-1}\omega - \tan^{-1}(\omega/2)$

$\because \quad \phi = -180^o \rightarrow \omega = \sqrt{2} \rightarrow (PCF)$ **ANS**

$$\rightarrow \quad G(s)H(s) = \frac{k}{\sqrt{2}\times\sqrt{(1+2)(4+2)}} = \frac{k}{6}$$

$GM = \frac{6}{k} = 2.5 \Rightarrow (k = 2.4)$ **ANS**

Sol.7(c) $$T(s) = \frac{10^8 (s+0.1)^3}{(s+10)^2 (s+100)}$$

$$\frac{k\left(1+\frac{s}{0.1}\right)^3}{\left(1+\frac{s}{10}\right)^2\left(\frac{s}{100}+1\right)} = \frac{k(s+0.1)^3\times10^3\times10^4}{(s+10)^2\times(s+10)} = \frac{k\times10^7(s+0.1)^3}{(s+10)^2(s+100)}$$ (here k=1) **ANS**